#Two sample t-test (for populations with UNEQUAL variance) prototype

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#A test for two sets of measurments from samples failing F-test for equal variances.

#Assumptions:

# Observed values x1.1...x1.n are a random sample from a normal distribution.

# Observed values x2.1...x2.n are a random sample from a normal distribution.

# Variances are unequal and unknown

# Both sample are independent.

## Note this test is reasonably robust for deviation from normal distribution.

# Hypotheses:

#1) Null: Mu1 is equal to Mu2.

#2) Alternative: Mu1 is not equal to Mu2 (two sided case)

# Mu1 < Mu2 (one sided case lower tail)

# Mu1 > Mu2 (one sided case upper tail)

#Paperwork

#read in data

iris

#assign variables

x1 <- iris$Sepal.Length[iris$Species=="setosa"]

x1

x2 <- iris$Sepal.Length[iris$Species=="versicolor"]

x2

#assign number of observations

n1 <- length(x1)

n1

n2 <- length(x2)

n2

#assign means

x1bar <- mean(x1)

x1bar

x2bar <- mean(x2)

x2bar

#assign standard deviations

s1 <- sqrt(var(x1))

s1

s2 <- sqrt(var(x2))

s2

#####Test Statistic#####

t <- (x1bar-x2bar)/sqrt((s1^2/n1)+(s2^2/n2)) #t is the normalized mean (x1bar-x2bar)

t

**[1] -10.52099**

#Satterthwaite's Method Degrees of Freedom:

df\_s <- (s1^2/n1+s2^2/n2)^2/(((s1^2/n1)^2/(n1-1))+(s2^2/n2)^2/(n2-1))

df\_s

**[1] 86.538**

#Sampling distribution: if assumptions hold and Null Hypothesis is true, then t~t(df\_s)

#Critical Values of the Test Note: two-tailed CVs are mirror values because of normal distribution

alpha <- 0.05 #probability of type 1 error

c1 <- qt(alpha/2, df\_s) #this is the two-sided lower critical value

c1

c2 <- qt(1-alpha/2, df\_s)#this is the two-sided higher critical value

c2

abs\_c <- abs(c1) #If using two-sided test, use absolute value of c1.

c3 <- qt(alpha, df\_s) #one sided case lower CV

c3

c4 <- qt(1-alpha, df\_s) #one sided case upper CV

c4

#Decision Rules:

#If abs(t) is > abs\_c, then reject Null, otherwise accept Null

#If t < c3, then reject Null \*\* one sided case lower tail

#If t > c4, then reject Null \*\* one sided case upper tail

#Probability (P) Value

p1 <- 2\*(pt(t, df\_s)) #two sided case, if t < or equal to 0

p1

**[1] 3.746743e-17**

p2 <- 2\*(1-pt(t, df\_s)) #two sided case, if t > 0

p2

p3 <- pt(t, df\_s) #one sided case lower tail

p3

p4 <- 1-pt(t, df\_s) #one sided case upper tail

p4

#Confidence Intervals for the Difference in Mean:

#two sided case = (ci\_a, ci\_b)

ci\_a <- x1bar-x2bar-abs\_c\*sqrt((s1^2/n1)+(s2^2/n2))

ci\_a

**[1] -1.105707**

ci\_b <- x1bar-x2bar+abs\_c\*sqrt((s1^2/n1)+(s2^2/n2))

ci\_b

**[1] -0.7542926**

ci\_l <- x1bar-x2bar-c3\*sqrt((s1^2/n1)+(s2^2/n2)) #one sided lower tail

ci\_l

ci\_u <- x1bar-x2bar-c4\*sqrt((s1^2/n1)+(s2^2/n2)) #one sided upper tail

ci\_u

# Now, test the built in R function

t.test(x1,x2,alternative = "two.sided",var.equal = FALSE, conf.level = 0.95)

**Results of Hypothesis Test**

**--------------------------**

**Null Hypothesis: difference in means = 0**

**Alternative Hypothesis: True difference in means is not equal to 0**

**Test Name: Welch Two Sample t-test**

**Estimated Parameter(s): mean of x = 5.006**

**mean of y = 5.936**

**Data: x1 and x2**

**Test Statistic: t = -10.52099**

**Test Statistic Parameter: df = 86.538**

**P-value: 3.746743e-17**

**95% Confidence Interval: LCL = -1.1057074**

**UCL = -0.7542926**